Chapter 1 Review Worksheet

Name: KEY Period: ____

Function Notation

- Relations vs. functions and the Vertical Line Test
- Function Composition - $f \circ g(x) = f(g(x))$

Given $f(x) = \frac{2x}{x+1}$ and $g(x) = 3x^2 - 2x$, evaluate or simplify the following:

1. $f(-3) = \frac{-6}{-2} = 3$

2. $f(-a) = \frac{-2a}{1-a}$

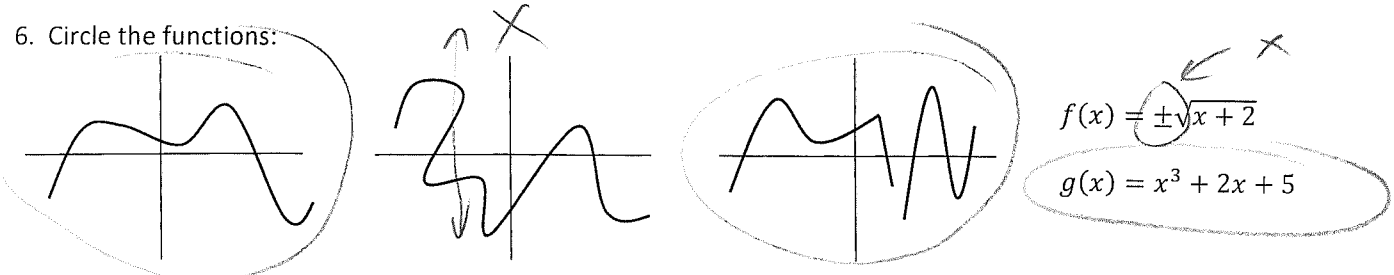
3. $f(x+1) = \frac{2x+2}{(x+1)+1} = \frac{2x+2}{x+2}$

4. $g(x-1) = 3(x-1)^2 - 2(x-1)$
$= 3x^2 - 6x + 3 - 2x + 2$
$= 3x^2 - 8x + 5$

4. $f(f(x)) = \frac{2\left(\frac{2x}{x+1}\right)}{\left(\frac{2x}{x+1}\right)+1} = \frac{\frac{4x}{x+1}}{\frac{2x}{x+1} + \frac{x+1}{x+1}}$
$= \frac{\frac{4x}{x+1}}{\frac{3x+1}{x+1}} = \frac{4x}{3x+1}$

5. $g(g(x)) =$
$3(3x^2-2x)^2 - 2(3x^2-2x)$
$3(9x^4 - 12x^3 + 4x^2) - 6x^2 + 4x$
$27x^4 - 36x^3 + 6x^2 + 4x$

6. Circle the functions:



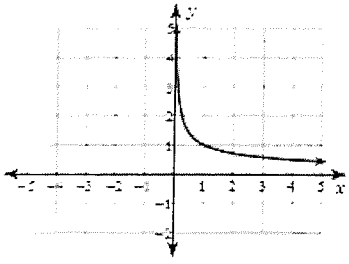
$f(x) = \pm\sqrt{x+2}$

$g(x) = x^3 + 2x + 5$

Domain/Range of f(x) using Interval Notation

State the domain and range of each function using interval notation.

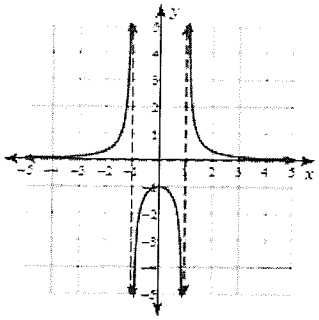
7. $f(x) = \frac{1}{\sqrt{x}}$



Domain: $(0, +\infty)$

Range: $(0, +\infty)$

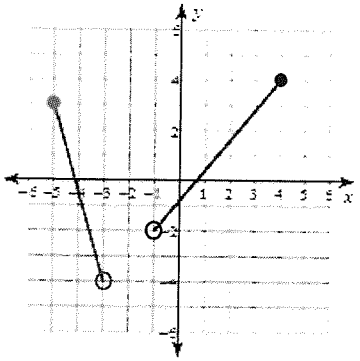
8. $f(x) = \frac{1}{x^2-1}$



Domain: $(-\infty, -1) \cup (-1, 1) \cup (1, +\infty)$

Range: $(-\infty, 1] \cup (0, \infty)$

9.



Domain: $[-5, -3) \cup (-2, 4]$ -1

Range: $(-4, 4]$

Function Transformation – Translations, Reflections, Stretches/Shrinks

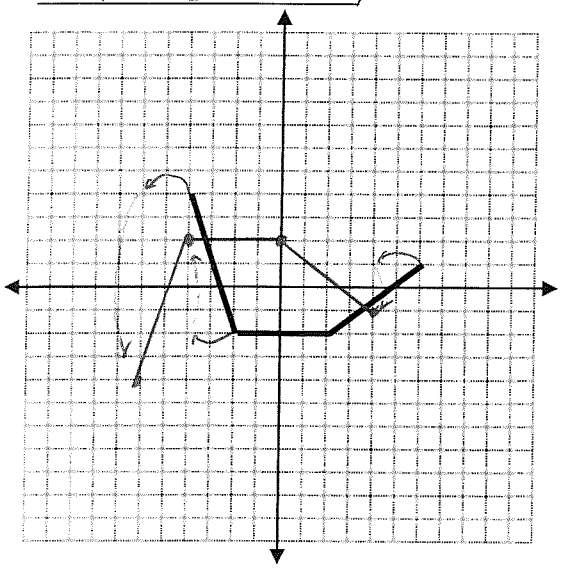
- Transform any function w/ translations, reflections and/or stretches/shrinks
- Graph transformations of functions
- Interpret transformations of a graph

In the following problems, $f(x)$ is given. Define and then graph the $t(x)$.

10. $t(x) = -f(x+2)$

Left by 2

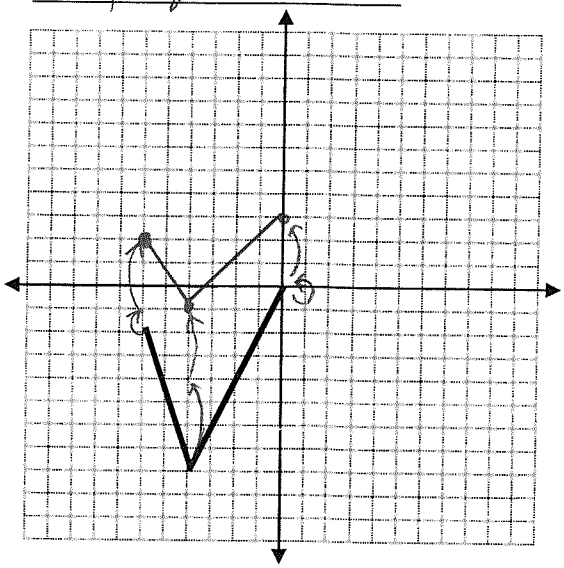
Reflect over $y=0$



11. $t(x) = \frac{1}{2}f(x) + 3$

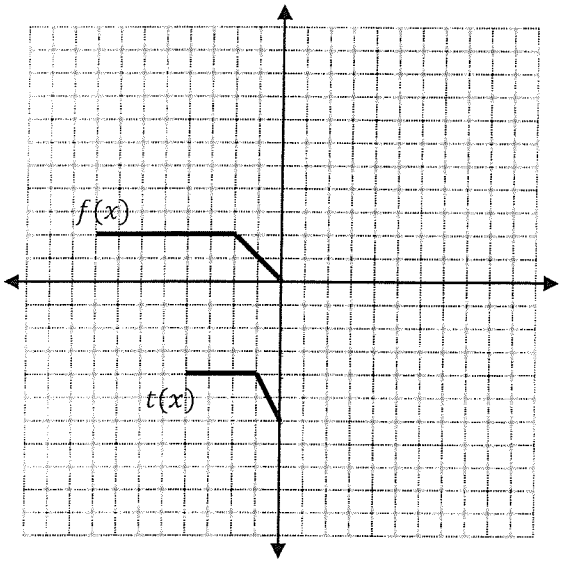
$\frac{1}{2}$ Vert. Shrink

up by 3



Given parent $f(x)$, state $t(x)$ in terms of $f(x)$.

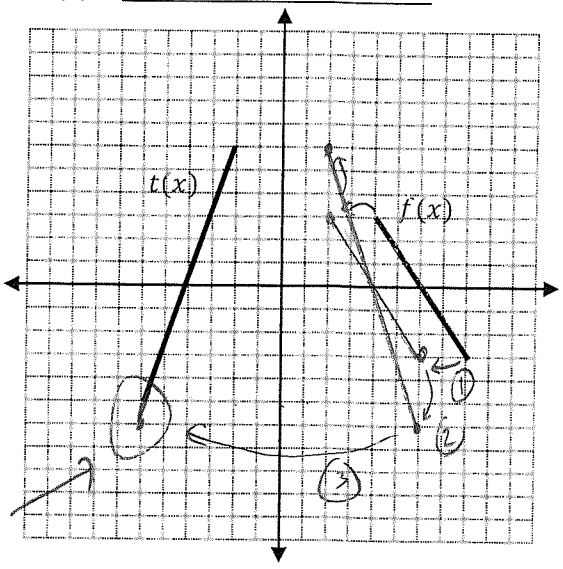
12. $t(x) =$ $f(2x)-6$



① Horizontal by $\frac{1}{2}$

② Down by 6

13. $t(x) =$ $2f(-x+2)$



FIX

① Left 2 $f(x+2)$

② VS 2 $2f(x+2)$

③ Refl over $x=0$ (y-axis) $2f(-x+2)$

14. Complete the following table given that $f(x)=x^2+1$:

	Function equation	Explicit equation
Shift up by 3	$g(x)=f(x)+3$	$g(x)=x^2+4$
Shift left by 2	$g(x)=f(x+2)$	$g(x)=(x+2)^2+1=x^2+4x+5$
Vertically stretch by a factor of 2	$g(x)=2f(x)$	$g(x)=2x^2+2$
Horizontally stretch by a factor of 2	$g(x)=f(\frac{1}{2}x)$	$g(x)=\frac{1}{4}x^2+1$
Reflect over the x-axis	$g(x)=-f(x)$	$g(x)=-x^2-1$
Reflect over the y-axis	$g(x)=f(-x)$	$g(x)=x^2+1$ even function...

15. Complete the following table given that $f(x)=|x+1|$:

	Function equation	Explicit equation
a) Shift down by 3	$g(x)=f(x)-3$	$g(x)=\|x+1\|-3$
b) Shift right by 2	$g(x)=f(x-2)$	$g(x)=\|x-1\|$
d) Horizontally compress by a factor of 0.5 and then a right shift by 3 (① $f_1(2x)$ ② $f_1(x-3)$)	$g(x)=f(2(x-3))$ $=f(2x-6)$	$g(x)=\|2x-5\|$
e) Reflect over the x-axis and then shift up by 2 (① $-f(x)$ ② $-f(x)+2$)	$g(x)=-f(x)+2$	$g(x)=-\|x+1\|+2$
f) Reflect over the y-axis and then shift right by 1 (① $f(-x)$ ② $f(-(x-1))$)	$g(x)=f(-x+1)$	$g(x)=\|-x+2\|$ ← ok $=\|x-2\|$ ← either
g) Shift right by 1 and then Reflect over the y-axis (① $f(x-1)$ ② $f(-x-1)$)	$g(x)=f(-x-1)$	$g(x)=\|-x\|=\|x\|$

16. Transform the point (3, 6) through the following transformations (one after the other).

a) Reflection over the x-axis: $(3,-6)$

b) Vertical stretch by factor of 2 $(3,-12)$

c) Left shift by 5 $(-2,-12)$

d) Horizontal shrink by factor 1/3 $(-\frac{2}{3},-12)$

17. Given <u>any</u> f(x) will the order of transformations matter if the two transformations are:

[Hint: Experiment with some points, or experiment with a simple function]

a) a shift up by 3 and a shift right by 7 — Yes / (No) ↑ →

b) a vertical stretch by 2 and a horizontal shrink by 2 — Yes / (No) ↕ ↔

c) a reflection over the x–axis and a shift up by 3 — (Yes) / No ↕ · ↕ +

d) a horizontal stretch by 2 and a reflection over the y-axis — Yes / (No) ↔ · ↔ ·

e) a vertical stretch by 3 and a shift up by 2 — (Yes) / No ↕ · ↕ +

Linear and Absolute Value Functions

- Generate line equations in slope-intercept or point-slope form
- Identify meaning of slope in context of graph
- Solve 2x2 and 3x3 system of linear equations

18. Write equation of line in point slope form. Explain meaning of the slope.

Equation: $(y-50) = -\frac{2}{5}(x-0)$

$y-50 = -\frac{2}{5}x$

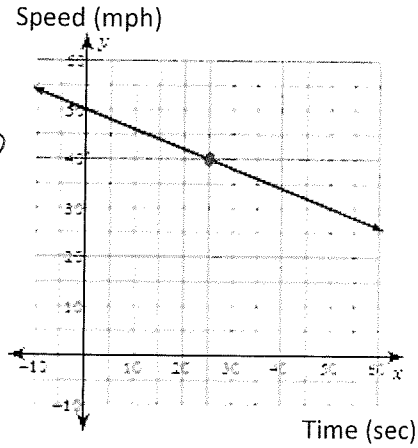
point: (0, 50)

$m = -\frac{2}{5}$

Slope meaning:

the rate of change of speed every second

→ "deceleration"



19. Write equation of line in slope-intercept form. Explain meaning of the slope.

Equation: ~~xxxxxxxxxxx~~

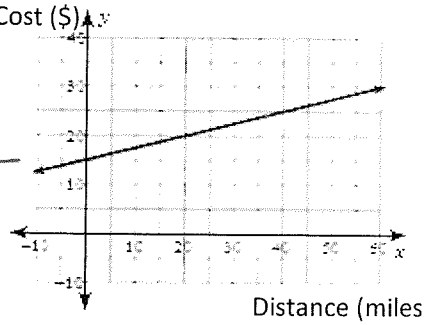
$y = 1/4\,x + 15$

m=1/4

~~xxxxxx~~

b = 15

Slope meaning: the additional cost to travel a mile



20. Solve: $6x + 6y - 3z = 6$ (If infinite solutions, write coordinate expression for solution space.)

① $4x + 3y - 5z = -9$

② $3x - 3y + 4z = 19$

③ → ④: $\div 2$ → $3x + 3y - \frac{3}{2}z = 3$

③ + ④: ⑥ $6x + \frac{5}{2}z = 22$

① − ④: $x - \frac{7}{2}z = -12$ ×6

⑤ − ⑥: ⑤: $6x - 21z = -72$

$(21 - \frac{5}{2})z = -94$

$-\frac{37}{2}z = -94$

$z = 4$

$x - \frac{7}{2}(4) = -12$

$x = -12 + 14$

$x = 2$

$4(2) + 3y - 5(4) = -9$

$3y = -9 + 20 - 8$

$y = 3$

$y = 1$

(2, 1, 4)

21. Solve: ① $-x + y + z = 3$

② $x + y + 3z = 5$

③ $3y + 6z = 12$

① + ②: $2y + 4z = 8$

$\div 2$ → $y + 2z = 4$

$\div 3$ → $y + 2z = 4$

→ infinite solutions

$y = 4 - 2z$

$x + (4 - 2z) + 3z = 5$

$x = 1 - z$

$z = 2 - \frac{1}{2}y$

$x = 1 - (2 - \frac{1}{2}y)$

$z = 1 - x$

$y = 4 - 2(1 - x)$

$= 2 + 2x$

(1 − z, 4 − 2z, z)

or

(x, 2 + 2x, 1 − x)

or

$(\frac{1}{2}y - 1, y, 2 - \frac{1}{2}y)$